

**CARDINAL HICKEY ACADEMY**  
**Summer MATH Packet**  
**Going into Grade 6**

**Study Guide**

**If you are unsure how to complete any of the problems in the summer packet here are some examples to help you.**

**Add Decimals: Regrouping**

Adding two decimal numbers with regrouping is very similar to adding whole numbers with regrouping. These problems are presented in a vertical format with the decimal points lined up correctly. These problems require regrouping, carrying, or trading.

A decimal number is a number that uses place value and a decimal point to show tenths, hundredths, thousandths, etc. For example, the number 3.57 has a 5 in the tenths place and a 7 in the hundredths place.

The difference between adding decimals with regrouping and adding whole numbers with regrouping is the fact that the decimal points must be lined up before addition can occur. The following is a step-by-step example of adding decimals with regrouping:

**Example 1:** Solve.  $2.4 + 1.68 = ?$

(1)	(2)	(3)	(4)	(5)	(6)
2.4	2.40	2.40	$\overset{1}{2}.40$	$\overset{1}{2}.40$	$\overset{1}{2}.40$
<u>+1.68</u>	<u>+1.68</u>	<u>+1.68</u>	<u>+1.68</u>	<u>+1.68</u>	<u>+1.68</u>
		8	08	.08	4.08

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Add one zero to 2.4 so that the two numbers have the same number of digits.

Step 3: Add the hundredths column ( $0 + 8 = 8$ ). Write the 8 in the hundredths column (below the line).

Step 4: Add the tenths column ( $4 + 6 = 10$ ). Write the 0 in the tenths column (below the line) and carry the 1 to the ones column (left of the decimal point).

Step 5: Bring the decimal point straight down.

Step 6: Add the ones column, including the 1 that was carried ( $1 + 2 + 1 = 4$ ). Place the 4 to the left of the decimal point to finish the problem.

The correct answer is  $2.4 + 1.68 = 4.08$ .

**Example 2:** Solve.  $3.2 + 2.8 = ?$

(1)	(2)	(3)	(4)
3.2	$\overset{1}{3}.2$	$\overset{1}{3}.2$	$\overset{1}{3}.2$
<u>+2.8</u>	<u>+2.8</u>	<u>+2.8</u>	<u>+2.8</u>
	0	.0	6.0

Step 1: Write the problem vertically. Make sure the decimal points are lined up.

Step 2: Add the tenths column ( $2 + 8 = 10$ ). Write the 0 in the tenths column (below the line) and carry the 1 to the ones column (left of the decimal point).

Step 3: Bring the decimal point straight down.

Step 4: Add the ones column, including the 1 that was carried ( $1 + 3 + 2 = 6$ ). Place the 6 to the left of the decimal point to finish the problem.

The correct answer is  $3.2 + 2.8 = 6.0$ .

### Add Decimals: Story Problems - A

Story problems, also called word problems, relate decimal numbers to actual situations. For example, if Sara weighs 32.5 pounds and Kimberly weighs 31.2 pounds, how much do Sara and Kimberly weigh together? The student must determine that addition is required to solve this problem. (Answer: 63.7 pounds)

Story problems are often very difficult for children to master. It may be beneficial for you to verify that the student is comfortable with addition and subtraction skills, as well as with reading skills. Relate word problems with everyday events.

**Example 1:** A recipe calls for 2.55 cups of flour and another recipe calls for 1.25 cups of flour. How many cups of flour do you need to make both recipes?

(1)	(2)	(3)	(4)	(5)
$\begin{array}{r} 2.55 \\ +1.25 \\ \hline \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 0 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 80 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline .80 \end{array}$	$\begin{array}{r} 2.55 \\ +1.25 \\ \hline 3.80 \end{array}$

**Step 1:** Rewrite the problem vertically. Always line up the decimal points.

**Step 2:** Add the numbers in the hundredths column ( $5 + 5 = 10$ ). Write the 0 in the hundredths position. Carry the 1 to the tenths column.

**Step 3:** Add the numbers in the tenths column, including the number carried over from the previous column ( $1 + 5 + 2 = 8$ ). Write the 8 in the tenths position.

**Step 4:** Bring the decimal point down.

**Step 5:** Add the numbers in the ones position ( $2 + 1 = 3$ ). Write the 3 to the left of the decimal point.

Answer: You need 3.80 cups of flour for the two recipes.

### Properties - A

Adding numbers in a different order helps students understand and apply the addition properties to larger numbers.

Numbers can be added in any order and the sum will not change. Help the student understand this by gathering together several different objects and adding them together in different combinations. For example, give the student 2 spoons, 3 shoes, and 5 pennies. Help him or her see that no matter which order you add the items the sum will remain the same. It doesn't matter if you add  $2 + 3 + 5$ ,  $5 + 3 + 2$ , or  $3 + 2 + 5$ , the sum will always be 10 items.

The addition properties are helpful in finding sums (the sum is the answer to an addition problem) and remembering addition facts.

The **Order Property for Addition** states that when the addend (addends are the numbers being added) order is changed, the sum is the same. When the student understands this concept, he or she will have fewer facts to remember.

**Example 1:**

$$5 + 4 = 9, \text{ so}$$
$$4 + 5 = 9$$

The **Zero Property for Addition** states that when one addend is 0, the sum is the other addend.

**Example 2:**

$$5 + 0 = 5, \text{ and}$$
$$0 + 5 = 5$$

The **Grouping Property for Addition** states that when the grouping of the addends is changed, the sum is the same.

**Example 3:**

$$\begin{array}{r} 5 \quad 3 \quad 2 \\ 3 \quad 2 \quad 5 \\ \hline 10 \quad 10 \quad 10 \end{array} \quad \begin{array}{l} \rightarrow \text{Either pair of numbers} \\ \text{can be added first, and} \\ \text{the sum is the same.} \end{array}$$

It may be helpful to write out some problems using the properties.

**Example 4:** What is the missing number in each mathematical sentence?

- (A)  $3 + \square = 5 + 3$
- (B)  $4 + 10 = 10 + \square$
- (C)  $\square + 11 = 11 + 4$

Solutions:

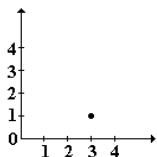
- (A). Answer: 5
- (B). Answer: 4
- (C). Answer: 4

**Coordinate Geometry - A**

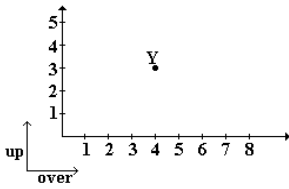
A coordinate graph is used to name the positions of objects placed on the graph.

It may be helpful to use graph paper to develop a coordinate graph. Help the student plot points on the graph and determine the coordinate pair.

**Example 1:** Plot a point that is 3 units over and 1 unit up from the zero. The coordinate points would be (3, 1). Remember that the horizontal position (or "over") is listed first in the coordinate pair, while the vertical position (or "up") is listed second.



**Example 2:** Where is the point Y?



Answer: Over 4, up 3 (always go over first, and then up)

### Add Decimals: Hundredths

Adding two decimal numbers with more than one digit (columns of numbers) is very similar to adding whole numbers. Like whole numbers, addition of decimals often requires regrouping (carrying, trading, renaming). Regrouping occurs when the total of the numbers in a column (i.e., ones position) is equal to or greater than ten. Problems are presented in both vertical and horizontal formats.

The following is a step-by-step example of a problem that requires regrouping:

**Example 1:** Solve.  $8.97 + 5.36 = ?$

$$\begin{array}{r}
 \text{(1)} \quad \text{(2)} \quad \text{(3)} \quad \text{(4)} \\
 \begin{array}{r}
 8.97 \\
 + 5.36 \\
 \hline
 \end{array}
 \end{array}$$

**Step 1:** Rewrite the problem vertically. Always line up the decimal points.

**Step 2:** Add the numbers in the hundredths position ( $7 + 6 = 13$ ). Write the 3 in the hundredths position. Carry the 1 to the next column (tenths).

**Step 3:** Add the numbers in the tenths column, including the number carried over from the previous column ( $1 + 9 + 3 = 13$ ). Write the 3 in the tenths position. Carry the 1 to the next column (ones). Bring the decimal point down.

**Step 4:** Add the numbers in the ones position, including the number carried over from the previous column ( $1 + 8 + 5 = 14$ ). Write the 14 to the left of the decimal point.

Answer:  $8.97 + 5.36 = 14.33$

**Example 2:** Solve.  $\$23.91 + \$32.64 = ?$

$$\begin{array}{r}
 \text{(1)} \quad \text{(2)} \quad \text{(3)} \quad \text{(4)} \quad \text{(5)} \quad \text{(6)} \\
 \begin{array}{r}
 \$23.91 \\
 + \$32.64 \\
 \hline
 \end{array}
 \end{array}$$

**Step 1:** Rewrite the problem vertically. Always line up the decimal points.

**Step 2:** Add the numbers in the hundredths position ( $1 + 4 = 5$ ). Write the 5 in the hundredths position.

**Step 3:** Add the numbers in the tenths column ( $9 + 6 = 15$ ). Write the 5 in the tenths position. Carry the 1 to the next column (ones).

**Step 4:** Bring the decimal point down.

**Step 5:** Add the numbers in the ones position, including the number carried over from the previous column ( $1 + 3 + 2 = 6$ ). Write the 6 in the ones position.

**Step 6:** Add the numbers in the tens position ( $2 + 3 = 5$ ). Write the 5 in the tens position. Bring down the dollar sign.

Answer:  $\$23.91 + \$32.64 = \$56.55$

### Add Fractions: Same Denominator

A fraction is comprised of two parts: a numerator (the top number) and a denominator (the bottom number). For example, in the fraction  $\frac{2}{3}$  the "2" is the numerator and the "3" is the denominator. Adding two fractions with the same denominator is the first step in calculating fractions.

The following is a step-by-step example of adding two fractions with the same denominator.

Solve:

$$\begin{array}{r} \frac{3}{8} + \frac{4}{8} = ? \\ \textcircled{1} \quad \textcircled{2} \\ \frac{3}{8} \quad \frac{3}{8} \\ + \frac{4}{8} \quad + \frac{4}{8} \\ \hline \frac{7}{8} \quad \frac{7}{8} \end{array}$$

**Step 1:** Rewrite the problem vertically.

**Step 2:** Add the numerators (the number above the line). The denominator (the number below the line) remains the same.

The correct answer is  $\frac{7}{8}$ .

### Identify Number Sentences - B

Students must determine what a given story problem is asking and then develop a formula for calculating how to solve the problem.

A creative method for improving the student's understanding of story problems is to develop humorous story problems and help the student solve the problems.

**Example 1:** 800 spaceships landed in your backyard on Monday. On Tuesday, 300 more spaceships landed in your backyard. How many spaceships landed in your backyard in all?

**Solution:** The student must determine that addition is needed to solve this problem. The number sentence is  $800 + 300 = 1,100$ .

Answer: 1,100 spaceships

**Example 2:** Seven boys and 13 girls went on the field trip. Which of these number sentences show how many children went on the field trip?

- A.  $13 \times 7$
- B.  $13 \div 7$
- C.  $13 - 7$
- D.  $13 + 7$

Answer: D

### Function/Pattern - A

Students must identify the equation that creates a specific pattern or function.

It may be helpful to develop a series of number patterns and help the student identify the pattern and determine the correct equation.

**Example:** The numbers in Column A have been changed to the numbers in Column B by using a specific rule.

A	B
3	1
9	3
12	4
15	5

Which number sentence shows that rule?

- A.  $A \times 3 = B$
- B.  $A - 3 = B$
- C.  $A + 3 = B$
- D.  $A \div 3 = B$

Solution: Since the numbers in Column B are less than Column A, we can conclude that Column B is either being subtracted or divided from Column A. Starting with the first row of numbers,  $3 - 3 = 0$ , so subtracting will not work. If we divide ( $3 \div 3 = 1$ ) it fits our pattern. Continue down the row to see if dividing works for all numbers in the pattern:

$$\begin{aligned} 9 \div 3 &= 3 \\ 12 \div 3 &= 4 \\ 15 \div 3 &= 5 \end{aligned}$$

Answer: D

**Example 2:** Use the given rule to complete the table.

Rule: Multiply by 2, then add 5

5	15
6	17
7	19
8	?

- (1)  $8 \times 2 = 16$
- (2)  $16 + 5 = 21$

Step 1: Multiply 8 by 2 to get 16.

Step 2: Add 5 to 16.

Answer: 21

### Averaging Numbers

The average of a group of numbers is found by dividing the sum of the numbers by the number of addends.

It may be useful to discuss with the student the common use of averaging numbers. Offer him or her examples, such as to find the average grade in a class. Help the student solve a series of averaging problems. The following is a step-by-step example of averaging numbers.

Find the average of 4, 5, and 6.

- (1)  $4 + 5 + 6 = 15$
- (2) 3 numbers to be averaged
- (3)  $15 \div 3 = 5$

Step 1: Add the numbers.

Step 2: Count the number of numbers to be averaged.

Step 3: Divide the sum (15) by the number of numbers (3).

Answer: The average is 5.

### Add Fractions: Different Denominator

A fraction is comprised of two parts: a numerator (the top number) and a denominator (the bottom number). For example, in the fraction  $2/3$ , the "2" is the numerator and the "3" is the denominator. In order to add fractions, you must have a common denominator. A common denominator is a whole number that is a common multiple of the denominators of two or more fractions. For example, 12 and 24 are both common denominators for  $3/4$  and  $5/6$ , because 4 and 6 will both divide into 12 and into 24.

The following is a step-by-step example of adding two fractions with different denominators.

**Example 1:** Reduce all fractions to lowest terms.

$$\frac{1}{5} + \frac{4}{9} = ?$$

(1)	(2)	(3)	(4)
$\frac{1}{5}$	$\frac{1 \times 9}{5 \times 9}$	$\frac{9}{45}$	$\frac{9}{45}$
$+\frac{4}{9}$	$+\frac{4 \times 5}{9 \times 5}$	$+\frac{20}{45}$	$+\frac{20}{45}$
			$\frac{29}{45}$

Step 1: Rewrite horizontal problems vertically. (This step is not necessary, but many students find it easier to add fractions when the problems are written vertically.)

Step 2: Find a common denominator (a common multiple of the denominators of two or more fractions). For this problem, the common denominator is 45, because 9 and 5 will both divide into 45.

Step 3: Multiply  $1/5$  by  $9/9$ . Rewrite the first fraction as  $9/45$ . Multiply  $4/9$  by  $5/5$ . Rewrite the second fraction as  $20/45$ .

Step 4: Add the numerators together ( $9 + 20 = 29$ ). The denominator (45) remains the same.

Answer:  $\frac{1}{5} + \frac{4}{9} = \frac{29}{45}$ .

It may be necessary to reduce a fraction that is part of an answer. A fraction is in lowest terms when the numerator and denominator do not have a common factor greater than one. To reduce a fraction, determine the largest number that the numerator and the denominator can both be divided by and divide them by that number.

**Example 2:** Reduce all fractions to lowest terms.

$$\frac{32}{40}$$

$$\frac{32 \div 8}{40 \div 8} = \frac{4}{5}$$

**Solution:** The largest number 32 and 40 can both be divided by is 8. Divide 32 by 8 and divide 40 by 8.

The correct answer is:  $\frac{32}{40}$  can be reduced to  $\frac{4}{5}$ .

An **improper fraction** is a fraction in which the numerator is greater than or equal to the denominator. All improper fractions can be rewritten as mixed numbers or as whole numbers. The following is an example of how to write an improper fraction as a mixed number.

**Example 3:** Reduce all fractions to lowest terms.

$$\frac{55}{18}$$

$$\frac{55}{18} = 3\frac{1}{18}$$

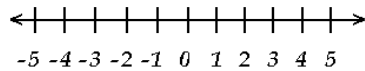
**Solution:** 18 will divide into 55 three times ( $18 \times 3 = 54$ ) with one left over. Write the 3 as a whole number and then make the fraction  $1/18$ .

The correct answer is:  $\frac{55}{18}$  can be written as  $3\frac{1}{18}$ .

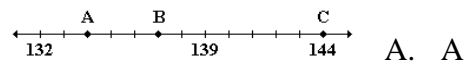
## Number Lines - B

A **number line** is a line with equally spaced points marked by numbers. Students are presented with problems which include calculating the sum and difference of points, as well as determining the value of a specific point.

An interesting method for improving the student's understanding of number lines is to develop a series of number lines. Help the student plot specific points on the number lines.



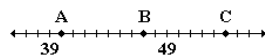
**Example 1:** Use the following number line to choose the point that is at 137.



- A. A  
B. B  
C. C  
D. none of the above

The answer is B.

**Example 2:** On the following number line, what is the difference between Point A and Point B?



- A. 8  
B. 7  
C. 10  
D. 6

**Solution:** There are two ways to find the answer. We can count over from A to B, (we would count 7).

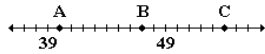


The answer is 7. Or, we can find the number of the points and subtract. Point A is at 40 and Point B is at 47.  $47 - 40 = 7$ .

The answer is 7.

**Example 3:** Use the number line to solve this equation.

$$\text{Point C} + 4 = ?$$



- A. 58
- B. 50
- C. 54
- D. 44

Solution: Count over from 49 to C. It is 54. Use 54 in the equation.  $54 + 4 = 58$

The answer is: A.

### Develop Story Problem: Equation

Story problems (word problems) require students to read a passage and determine the question being asked. They should then identify the elements needed to solve each problem, decide on the correct method to solve the problem, and find a solution. For this skill, the student must choose the story problem that represents the mathematical symbols given, but he or she is not required to find the solution. Each of the questions in this skill contain only one operation. The student's textbook may have word problems that he or she can use to practice the steps involved in setting up mathematical problems from text.

#### Example 1:

Which of the following word problems can be solved using the equation  $44 - 5 = x$ ?

- A. The movie theater sold 44 tickets for the matinee showing. If the tickets cost 5 dollars each, how much money did they collect in ticket sales for that showing?
- B. The butterfly exhibit at the zoo contains 44 different species. After a trade with another zoo, they received 5 new species of butterflies. How many different species does the zoo have now?
- C. In the past 44 days in Seattle, it was sunny for 5 days and rained all of the rest. How many days did it rain?
- D. The hot water tank holds 44 minutes worth of hot water for showers. If 5 people need to take a shower, how long can each of their showers last so that no one runs out of hot water?

**Answer:** (C).

(A) is not correct. If each person is paying 5 dollars and there are 44 people, determining the total amount of money collected is a multiplication problem. This would result in  $44 \times 5 = x$ .

(B) is not correct. The zoo is increasing its number of butterfly species by receiving 5 new ones. This is an addition problem, and the equation would be  $44 + 5 = x$ .

(C) is the correct answer. To determine the number of days that it rained, we must take the total of 44 days and subtract the 5 sunny days, giving us  $44 - 5 = x$ .

(D) is not correct. The 44 minutes is split equally among 5 people. Each person gets a portion of the time, so this is a division problem and the equation would be  $44 \div 5 = x$ .

### Example 2:

Which of the following word problems could be solved using the equation  $18 \times 3 = x$ ?

A. Early in the evening, there were 18 people at Bob's house for a barbecue. Later on, 3 more people came. How many people attended Bob's barbecue?

B. Rodolfo bought a carton of 18 eggs. After he used 3 eggs to bake a cake, how many eggs were left in the carton?

C. There are 18 tennis balls left in the bucket. If 3 players are using the balls, how many tennis balls can each of them take?

D. Ms. Iwasaki needs to copy the test she is giving to her 18 students tomorrow. If the test is 3 pages long, how many pieces of paper will she use?

**Answer:** (D).

(A) is not correct. There were 18 people at the party and then 3 more were added to the total number who attended. This is an addition problem, and the equation would be  $18 + 3 = x$ .

(B) is not correct. Rodolfo started with 18 eggs and then removed 3 eggs. This is a subtraction problem, and the equation would be  $18 - 3 = x$ .

(C) is not correct. The 18 tennis balls are split equally among 3 people. Each person gets his or her share of the tennis balls. This is a division problem, and the equation would be  $18 \div 3 = x$ .

(D) is the correct answer. Each of the 18 students gets the 3-page test. This is a multiplication problem, and the equation would be  $18 \times 3 = x$ .

An activity that will reinforce this skill is to have a single student or a group of students work to come up with examples that represent the four different operations using the same numbers. Give the students two numbers, for example 15 and 6, and have them write one addition, one subtraction, one multiplication, and one division story problem that uses the two numbers.

### Experimental Probability

Probability is the ratio of the number of times a certain outcome can occur to the number of total possible outcomes. The probability of an event cannot be smaller than zero or larger than one. A probability of zero means there is no chance of an event occurring and a probability of one means that an event is certain to occur.

**Example 1:**

If there are 6 green marbles, 3 orange marbles, 2 blue marbles, and 1 black marble in a bag, what is the probability that an orange marble will be blindly pulled from the bag first?

**Solution:**

Start with the number of marbles in the bag. There is a total of 12. Then, figure out how many marbles are orange. There are 3. The probability ratio is  $3/12$ , or, in reduced form,  $1/4$ .

**Answer:**  $\frac{1}{4}$

At times, experimental probability questions involve working with a standard deck of cards. Therefore, it is important for the student to know the composition of a deck of cards. A standard deck contains fifty-two cards. It also contains four different suits: spades, clubs, diamonds, and hearts. Each suit has thirteen cards. The suits of diamonds and hearts are red cards and the suits of spades and clubs are black cards.

**Example 2:**

If you were to draw a playing card from a standard deck of 52 cards, what is the probability of drawing a 3 of diamonds?

**Solution:**

Since there are 52 cards in the deck, and there is only one 3 of diamonds, the probability is  $1/52$ .

**Answer:**  $\frac{1}{52}$

**Experimental Probability** is found by gathering data through observation or experimentation. The ratio to determine experimental probability is:

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of times the experiment is performed}}$$

Experimental probability is not determined using theoretical data. It is strictly the result of the experiments that are performed. For example, if a coin is flipped, the mathematical probability of getting heads is  $\frac{1}{2}$ , as it is equally likely to get heads or tails. It is unlikely that if an actual experiment was performed, the results will come out to be exactly  $\frac{1}{2}$ . If a coin is flipped ten times, it might land on heads seven times, not five as would be expected mathematically. It is important to remember that the more times an experiment is performed, the closer the result will be to the true mathematical (called theoretical) probability.

**Example 3:**

Jill was playing cards with her friends and was dealt two pairs twice during the thirteen games that they played. Based on this information, what is the experimental probability that Jill will be dealt two pairs on the next hand?

**Solution:**

The experimental probability is the ratio of the number of times Jill was given two pairs, 2, to the number of times she played the game, 13.

**Answer:**  $\frac{2}{13}$

**Example 4:**

Joon kept track of the minutes that he worked late at his job last month in the following chart.

Number of Minutes	Number of Times
0	5
15	8
30	5
45	2
60	1

Based on this information, what is the experimental probability that he will work 30 minutes late the next day he has to work?

**Solution:**

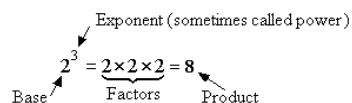
The experimental probability is the ratio of the number of times Joon worked 30 minutes late, 5, to the total number of days when he was keeping track of his hours, 21.

**Answer:**  $\frac{5}{21}$

An activity that may help to reinforce this skill could be to buy a large bag of some type of different colored, small candies. Empty the candy into a large paper bag. Have the students choose their favorite color of those candies. Then have them pick out 10 candies and count out the number of candies of that color and the total number of candies that they pull out. From this information, help them determine the experimental probability that the next candy that they pull out will be their favorite color. Replace the candy into the bag. Repeat the experiment two more times, the next time pulling out 5 candies and the third time pulling out 20 candies.

**Calculating With Exponents**

In this study guide, students will learn how to perform basic calculations with exponents. An exponent is a number that represents repeated multiplication. It tells the student how many times the base is used as a factor. A factor is a number that is multiplied by another number. For example, the base number 2 with an exponent of 3 is equal to  $2 \times 2 \times 2$ . It is usually written in the following format:



Before calculating exponents within an expression, find the equivalent whole number forms of these exponential numbers:

$6^2$  (or "six to the second power" or "six squared")  
 $6 \times 6 = 36$

$3^3$  (or "three to the third power" or "three cubed")  
 $3 \times 3 \times 3 = 27$

$2^6$  (or "two to the sixth power")  
 $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

The most common error among students learning about exponents is to multiply the base number by the exponent. That is, many students will calculate 8 to the 3rd power as  $8 \times 3 = 24$ , instead of  $8 \times 8 \times 8 = 512$ .

**Calculating Exponents Within An Expression**

When working with exponents within an expression, the student must remember the rules for the order of operations. The order of operations can be remembered with the phrase "Please Excuse My Dear Aunt Sally."

The student should always perform operations in the following order:

**P** - Parentheses

**E** - Exponents

**M/D** - Multiplication/Division in order from left to right

**A/S** - Addition/Subtraction in order from left to right

**Example 1:** Subtract.

$$5,000 - 8^4 =$$

(1)  $8^4 = 8 \times 8 \times 8 \times 8 = 4,096$

(2)  $5,000 - 4,096 = 904$

Step 1: According to the rules for order of operations, calculate the exponent first.

Step 2: Subtract 4,096 from 5,000.

**Answer:**  $5,000 - 8^4 = 904$

**Example 2:** Add.

$$9^2 + 3^4 =$$

(1)  $9 \times 9 = 81$

(2)  $3 \times 3 \times 3 \times 3 = 81$

(3)  $81 + 81 = 162$

Step 1: Since both terms in the expression contain exponents, the student should work from left to right. Calculate  $9^2$  first.

Step 2: Next, calculate  $3^4$ .

Step 3: Finally, add  $81 + 81$ .

**Answer:**  $9^2 + 3^4 = 162$

To help the student practice calculating exponents try the following activity. Begin with several blank index cards. First, write the numbers 1-10 on ten different cards. Next, write *exponent of 1*, *exponent of 2*...up to *exponent of 10* on ten different cards. Then, write ten whole numbers of your choice on ten different cards. Finally, write the addition and subtraction symbols on two different cards. (For a challenge you may want to include multiplication and division.) Sort the cards into 4 separate piles (single-digit whole numbers, exponents, whole numbers, and operational symbols). Turn all the cards face down. Have the student randomly select one card from each pile and then combine the numbers and symbols to create an expression. Finally, have the student calculate the various exponential expressions.

### **Exponential Notation - A**

An exponent is a number that represents how many times the base is used as a factor. For example, the number 2 with an exponent of 3 is equal to  $2 \times 2 \times 2$ .

Have the student find the equivalent whole number forms of these exponential numbers:

$$\begin{array}{l} \textcircled{1} \\ 6^2 \text{ (or 6 to the second power)} \\ 6 \times 6 = 36 \\ \textcircled{2} \\ 3^3 \text{ (or 3 to the third power)} \\ 3 \times 3 \times 3 = 27 \\ \textcircled{3} \\ 2^5 \text{ (or 2 to the fifth power)} \\ 2 \times 2 \times 2 \times 2 \times 2 = 32 \end{array}$$

The most common error among students learning about exponents is multiplying the base number by the exponent. That is, many students will calculate 8 to the 3rd power as  $8 \times 3 = 24$ . The correct answer is  $8 \times 8 \times 8 = 512$ .

### **Rounding and Estimation - C**

Rounding and estimation are used to express a number to the nearest ten, hundred, thousand, and so forth.

An interesting method for improving the student's rounding and estimation skills is to create a list of numbers. Help the student round each number. Remember, numbers less than 5 are rounded down, while numbers 5 or greater are rounded up (in both cases, you are looking one place to the right of the place value you wish to round).

**34 rounded to the nearest ten is 30.**

**37 rounded to the nearest ten is 40.**

In order to round decimal numbers to whole numbers, we look at the digit in the tenths place. If the digit is less than 5, drop the decimal part. If the digit is 5 or more, drop the decimal part and round up.

#### **Example 1:**

7.328 rounded to the nearest whole number is 7

8.74 rounded to the nearest whole number is 9

In estimating an answer, we round the numbers we are operating with in order to determine a simpler answer. The examples below illustrate this process.

**Example 2:** Which of the following formulas should be used to estimate  $36.3 \times 8.9$ ?

- A.  $37 \times 8$
- B.  $37 \times 9$
- C.  $36 \times 9$
- D.  $36 \times 8$

**Solution:** Round both numbers to the nearest whole number.

36.3 rounds to 36

8.9 rounds to 9

The answer is C.

**Example 3:** Which of the following formulas should you use to estimate  $7,849 \times 3,434$ ?

- A. 7,000 x 3000
- B. 7,000 x 4000
- C. 8,000 x 3,000
- D. 8,000 x 4000

Solution: We round both numbers to the nearest thousand.

7,849 rounds to 8,000

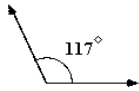
3,434 rounds to 3,000

The answer is C.

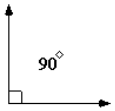
### Angles - A

An angle consists of two rays with the same endpoint. The endpoint is called the vertex of the angle.

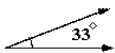
An interesting method for improving the student's understanding of angles is to have him or her draw the various types of angles on a series of flash cards. On the other side of the card, write the name. The following definitions will help you get started.



Obtuse Angle - an angle with a measure greater than 90 degrees and less than 180 degrees



Right Angle - an angle with a measure equal to 90 degrees

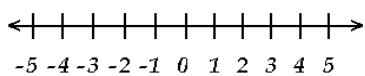


Acute Angle - an angle with a measure greater than 0 degrees and less than 90 degrees

### Adding Integers

Integers are the set of positive and negative whole numbers, including zero. To add integers, students must understand how integers appear on a number line. Numbers to the right of 0 on a number line are positive and numbers to the left of 0 are negative. The number -3 is a negative integer and the number 3 is a positive integer. The number zero is neither positive nor negative, it is neutral.

It may be beneficial to verify that the student understands integers by having him or her create a number line. Label points to the left of 0 "negative," and points to the right of the 0 "positive." The following is an example of a number line:



Confirm that the student understands that -4 is less than 4. Once he or she is comfortable with the concept of integers, introduce adding and subtracting. For example,  $-4 + 2$ . Start at -4 and move 2 places to the right (because we are adding). The answer is -2.

When adding two integers with the same sign, add their absolute values. Then give the sum (answer) the sign of the integers.

$$\begin{aligned} -3 + -2 &= ? \\ |-3| + |-2| &= ? \\ 3 + 2 &= 5, \text{ then make the result negative.} \end{aligned}$$

Answer: -5

When adding integers with different signs, first find their absolute values. Then subtract the lesser absolute value from the greater absolute value, and give the result the sign of the integer with the greater absolute value.

$$\begin{aligned} -7 + 3 &= ? \\ |-7| = 7 \text{ and } |3| = 3 \text{ (find the absolute values)} \\ 7 - 3 &= ? \text{ (subtract the lesser from the greater)} \\ 7 - 3 &= 4 \\ -7 + 3 &= -4 \text{ (The result is given the sign of the greater integer.)} \end{aligned}$$

## Comparison

Comparisons at this grade level involve whole numbers, fractions, decimals, percents, and integers. Students must compare the value of given equations.

It may be necessary to review integers, decimals, fractions, and percents with the student. Help the student understand that integers include whole numbers, their opposites, and zero. Practice integers using a number line by plotting points such as -5, -1, 0, and 2.

As the student practices comparisons, remind him or her that fractions, percentages, and decimals represent portions or parts and that for every fraction, percentage, or decimal, there is a corresponding portion. The fraction  $\frac{1}{2}$  communicates a specific portion of something, but this specific portion can also be communicated by the percentage 50%. Decimals are portions communicated in columns (place values) which represent an unwritten denominator of 10 or a power of 10.  $3.23$  expresses 3 wholes and 23 hundredths of a whole.

To compare fractions, percentages, and decimals, they must be converted to the same form. The fraction  $\frac{3}{4}$  can be compared to the percentage 60% by converting the percentage to fraction form. Percentage means "per one hundred," so  $60\% = \frac{60}{100}$ . Find a common denominator between  $\frac{60}{100}$  and  $\frac{3}{4}$ . 100 can be divided by 4, so the fractions become:  $\frac{60}{100}$  compared to  $\frac{75}{100}$ . From this comparison we can see that  $\frac{75}{100}$  or  $\frac{3}{4}$  represents a greater portion than  $\frac{60}{100}$  or 60%.

To compare the decimal 0.74 to the fraction  $\frac{3}{4}$ , convert the decimal to fraction form. 0.74 becomes  $\frac{74}{100}$  or  $\frac{74}{100}$ . We already know that  $\frac{3}{4} = \frac{75}{100}$ . Therefore,  $\frac{3}{4}$  represents a greater portion than 0.74 or  $\frac{74}{100}$ .

## Ratio/Proportion - B

A ratio is a comparison of two numbers expressed as a quotient. They can be written in three ways: a fraction ( $\frac{3}{5}$ ), a ratio (3:5), or a phrase (3 to 5). Like fractions, ratios refer to a specific comparison. The ratios  $\frac{3}{5}$ , 3:5, and 3 to 5 (as in "the ratio of cellos to violins was 3 to 5") all express the same ratio or comparison. A



proportion reflects the equivalency of two ratios. The ratio  $\frac{3}{5}$  expresses the same proportion as the ratio  $\frac{15}{25}$ .

To understand how ratios operate, students need to understand equivalent fractions. Fractions represent portions or parts. For every fraction, there is a corresponding portion. The fraction  $\frac{1}{2}$  communicates a specific portion of something, but this specific portion can also be communicated by the fractions  $\frac{2}{4}$ ,  $\frac{3}{6}$ ,  $\frac{8}{16}$ ,  $\frac{10}{20}$ , etc. All of these fractions are equal to  $\frac{1}{2}$  because the relationship between the numerator and denominator in  $\frac{1}{2}$  is the same relationship between the numerators and denominators in  $\frac{2}{4}$ ,  $\frac{3}{6}$ ,  $\frac{8}{16}$ , and  $\frac{10}{20}$ . Ratios and proportions operate in a similar manner. The ratio 2:5 communicates a specific portion. The ratio 4:10 communicates the same portion.

**Example 1:** Sandra has 15 lollipops and 25 jellybeans. What is the ratio of lollipops to jellybeans?

There are 15 lollipops to 25 jellybeans, so the ratio of lollipops to jellybeans is 15:25.

Answer: 15:25

Proportions occur when two ratios are equal. In a proportion the cross products of the terms are equal.

**Example 2:** Is the following proportion True or False?

$$\frac{1}{3} = \frac{3}{9}$$

$$\begin{array}{r} \frac{1}{3} \times \frac{3}{9} \\ \hline 9 \times 1 = 3 \times 3 \\ 9 = 9 \end{array}$$

The cross products are both equal to 9, so the proportion is TRUE. If the cross products are not equal, the proportion is false.

Sometimes you must find the value of a variable in a proportion. To solve the proportion, you must find the value of the variable that makes both ratios equal.

**Example 3:**  $\frac{9}{12} = \frac{a}{48}$

$$\begin{array}{r} \frac{9}{12} \times \frac{a}{48} \\ \hline (1) 48 \times 9 = 12 \times a \\ (2) 432 = 12a \\ (3) \frac{432}{12} = \frac{12a}{12} \\ (4) 36 = a \end{array}$$

Step 1: Find the cross products. Multiply 48 by 9 and 12 by 'a'.

Step 2:  $48 \times 9 = 432$  and  $12 \times a = 12a$ . Rewrite the equation with the new products.

Step 3: Divide each side of the equation by 12 to isolate the variable 'a'.

Step 4: Divide 432 by 12 to get  $a = 36$ .

Answer:  $a = 36$

### **Expressions: Addition**

Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.

Example:  $y - 6$

An equation is a number sentence that does have an equal sign.

Example:  $y - 6 = 14$

**Example 1:** Evaluate the expression  $x + 23$ , when  $x = 5$ .

$$5 + 23$$

Solution: Substitute the value 5 in place of x in the expression.

Answer: 28

**Example 2:** For  $x = -7$ , find  $2x + -12$ .

(1)  $2(-7) + -12$

(2)  $-14 + -12$

(3)  $-26$

Step 1: Substitute -7 in for the value of x.

Step 2: Multiply 2 by -7 and rewrite the expression with the new value.

Step 3: Add -14 and -12.

Answer: -26

**Example 3:** Write a mathematical expression to represent the following:

The sum of a number and 23.

Solution: Remember that "sum" is the answer to an addition problem, so the expression is  $x + 23$ .

Answer:  $x + 23$

### **Expressions: Multiplication**

Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.

Example:  $y - 6$

An equation is a number sentence that does have an equal sign.

Example:  $y - 6 = 14$

**Example 1:** Evaluate the expression below for  $x = 18$ .

$$5 + 3x$$

(1)  $5 + 3(18)$

(2)  $5 + 54$

(3) 59

Step 1: Replace x with 18.

Step 2: Multiply 3 by 18 to get 54.

Step 3: Add 5 and 54.

Answer: 59

**Example 2:** Write a mathematical expression that represents the following word expression.

four times a number less 6

- (1) four times x less 6
- (2)  $4x$  less 6
- (3)  $4x - 6$

Step 1: Replace the words "a number" with a variable (x was chosen)

Step 2: "four times x" can be written as  $4x$ . Make this replacement.

Step 3: "less 6" can be written as "- 6." Make this replacement and the expression is complete.

Answer:  $4x - 6$

### **Dividing Integers**

Integers are the set of positive and negative whole numbers, including zero. To find the quotient (answer to a division problem) of two integers, the following rules apply:

The quotient of two integers with different signs is negative. Example:  $16 \div -4 = -4$ .

The quotient of two integers with the same sign is positive. Examples:  $16 \div 4 = 4$  and  $-16 \div -4 = 4$ .

Operations within parentheses are completed first. After performing operations within parentheses, perform all multiplication and division in order from left to right. The last step is to perform all addition and subtraction in order from left to right. (It may be helpful here to review order of operations and/or multiplying with integers.)

**Example 1:**  $4(-3 \times 2) \div (12 \div 2) = ?$

- (1)  $-3 \times 2 = -6$  and  $12 \div 2 = 6$
- (2)  $4(-6) \div 6 = ?$
- (3)  $-24 \div 6 = ?$
- (4)  $-24 \div 6 = -4$

Step 1: Perform operations within parentheses: ( $-3 \times 2 = -6$ ) and ( $12 \div 2 = 6$ ).

Step 2: Write out the problem, replacing the values within the parentheses with the new values.

Step 3: Perform multiplication or division in order from left to right. Multiply first because it comes first when reading from left to right.  $4(-6) = -24$ .

Step 4: Divide -24 by 6 to get -4. Remember the quotient of two integers with different signs is negative.

Answer: -4

The following example illustrates the use of rules for dividing integers using "is greater than" ( $>$ ) and "is less than" ( $<$ ).

**Example 2:**  $-24 \stackrel{?}{>} 3(10 \div -2)$

- (1)  $10 \div -2 = -5$
- (2)  $-24 \div 3(-5)$
- (3)  $3 \times -5 = -15$
- (4)  $-24 \div -15$
- (5)  $-24 < -15$

**Step 1:** Perform operations within parentheses.  $10 \div -2 = -5$ .

**Step 2:** Rewrite the problem with -5 in place of the parentheses.

**Step 3:** Multiply  $3 \times -5$  to get -15.

**Step 4:** Rewrite the problem with -15 in place of  $3(-5)$ .

**Step 5:** To determine which symbol to place between -24 and -15, think of the integers as being money. -24 would be like owing someone \$24 and -15 would be like owing someone \$15. Since owing \$24 is more in debt than owing \$15, -24 is less than -15.

Answer:  $-24 < -15$

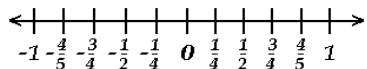
### Addition/Subtraction Rational Numbers

The following numbers are rational numbers because they can all be written as fractions.

$$\begin{array}{ll} 0.6 = \frac{3}{5} & 0 = \frac{0}{1} \\ -29 = \frac{-29}{1} & 2 = \frac{2}{1} \end{array}$$

Adding and subtracting rational numbers includes the calculation of whole numbers, fractions, decimals, and integers.

The most common error that students make when learning how to add and subtract rational numbers is incorrectly ordering the rational numbers. A useful technique for showing the student how to identify rational numbers is to create a number line. Negative numbers appear to the left of 0, while positive numbers appear to the right. The following is an example of a number line:



Confirm that the student understands that for any two rational numbers on the number line, the number to the right is always the greater number. For example,  $1/4$  is greater than  $-1/4$ . When he or she understands how to order rational numbers, have him or her add  $1/4$  to  $-1/4$ . The answer is 0 as shown in the number line above because  $-1/4$  is  $1/4$  places from 0 on the number line.

**Example 1:**  $-3.2 + -5.7 - -4.3 = ?$

- (1)  $-3.2 + -5.7 = -8.9$
- (2)  $-8.9 - -4.3 = -8.9 + 4.3 = -4.6$

**Step 1:** Add or subtract the first two numbers from left to right. Remember that when we add two negative numbers, we add their absolute values together and give the result a negative sign.

**Step 2:** Replace  $-3.2 + -5.7$  with -8.9 and continue adding or subtracting left to right. Remember that when we subtract a negative, it's the same as adding a positive, so we can rewrite  $-8.9 - -4.3 = ?$ , as  $-8.9 + 4.3 = ?$

Answer: -4.6

**Example 2:**  $2 \frac{1}{2} - 3 \frac{3}{4} + -5 \frac{5}{8} =$

There are several ways to solve this problem with fractions, one method is to change the fractions to improper fractions:

$$2 \frac{1}{2} = \frac{5}{2}$$

$$3 \frac{3}{4} = \frac{15}{4}$$

$$5 \frac{5}{8} = \frac{45}{8}$$

Then we can find a common denominator and perform the addition and subtraction. The common denominator is 8, so we make each denominator 8:

$$\frac{5}{2} = \frac{20}{8}$$

$$\frac{15}{4} = \frac{30}{8}$$

$$\frac{45}{8} = \frac{45}{8}$$

Now we can rewrite the problem as:

$$\frac{20}{8} - \frac{30}{8} + \frac{-45}{8} = ?$$

$$(1) \frac{20}{8} - \frac{30}{8} = \frac{20}{8} + \frac{30}{8} = \frac{50}{8}$$

$$(2) \frac{50}{8} + \frac{-45}{8} = \frac{5}{8}$$

Step 1: Add or subtract the first pair of fractions from left to right.

Step 2: Replace  $\frac{20}{8} - \frac{30}{8}$  with  $\frac{50}{8}$  and continue adding or subtracting from left to right.

Answer:  $\frac{5}{8}$

### **Expressions: Subtraction**

Expressions look like equations except expressions do not have equal (=) signs. Students don't "solve" an expression, they "evaluate" or "simplify" it.

The following are step-by-step examples of how to evaluate expressions.

**Example 1:** Evaluate the expression for  $n = -2$ .

$$n - 6$$

Solution: Substitute -2 in place of n and simplify.

$$-2 - 6 = -8$$

Answer: -8

**Example 2:** Evaluate the expression for  $x = 10$ .

$$35 - x$$

Solution: Substitute 10 in place of x and simplify.

$$35 - 10 = 25$$

Answer: 25

**Example 3:** What mathematical expression best represents the word expression "a number decreased by 12"?

Solution: We can represent a number with any variable, we'll use t. The number is decreased by 12, so we subtract.

Answer:  $t - 12$

### **Expressions: Division**

Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.

Example:  $y - 6$

**Example 1:** Evaluate the following expression when  $n = -3$ .

$$\frac{2}{n}$$

(1)  $2/(-3)$   
(2)  $-2/3$

Step 1: Substitute the value of n into the expression.

Step 2: Leave the expression in fraction form and place the negative sign in front.

Answer: For the value of  $n = -3$ , the expression is equal to  $-2/3$ .

**Example 2:** Evaluate the expression for  $n = -3$ .

$$\frac{(n-7)}{2}$$

(1)  $(-3 - 7)/2$   
(2)  $-10/2$   
(3)  $-5$

Step 1: Substitute the value of n into the expression.

Step 2: Simplify  $-3 - 7$  to get  $-10$ .

Step 3: Divide  $-10$  by  $2$  to get  $-5$ .

Answer: For the value of  $n = -3$ , the expression is equal to  $-5$ .

**Example 3:** Which mathematical expression best represents the word expression?

The amount of wheat, w, divided by 6 people.

Use the variable, w, and show division by 6.

Answer:  $w/6$  or  $w \div 6$

## Tables - F

Tables are visual aids used to express information.

The following is a table displaying the different prices for stickers, ink pads, and stamps at three different stores: Store A, Store B, and Store C.

	Stickers	Ink Pads	Stamps
Store A	15 for \$1.00	\$2.50 each	2 for \$5.50
Store B	10 for \$.95	3 for \$8.00	\$3.00 each
Store C	13 for \$.65	4 for \$9.00	3 for \$9.75

**Example 2:** Use the table above to answer the question. At Store C, how much would one ink pad cost?

### Solution:

Select the item (ink pad) from the top row and the store (Store C) from the left column. Follow the ink pad column down and the store row across until the two meet. You find "4 for \$9.00". To calculate the cost of one ink pad, divide \$9.00 by 4.

**Answer:** \$2.25.

**Example 3:** Use the table above to answer the question. Which store is the least expensive for one sticker?

### Solution:

(1) Select the item (stickers) from the top row.

(2) For Store A, stickers are sold 15 for \$1.00. To find the price of one sticker, divide \$1.00 by 15. The result is 6.7 cents each.

(3) For Store B, stickers are sold 10 for \$0.95. To find the price of one sticker, divide \$0.95 by 10. The result is 9.5 cents each.

(4) For Store C, stickers are sold 13 for \$0.65. To find the price of one sticker, divide \$0.65 by 13. The result is 5 cents each.

Stickers at Store A are 6.7¢ each.

Stickers at Store B are 9.5¢ each.

Stickers at Store C are 5¢ each.

**Answer:** Store C has the least expensive stickers.

## Predictions

Probability is the measure of the chance that a specific outcome will occur. When the probability of an event is known, it is possible to predict the expected outcome for that event. When the probability of an event is based on experience, it is called empirical or experimental probability.

Students must first understand the formula used for solving problems with experimental or empirical probability. In the following equation,  $n$  stands for the number of trials,  $e$  stands for the number of times an event occurred, and  $P(e)$  stands for the experimental or empirical probability:  $P(e) = e/n$ .

**Example 1:** Predict the results of Jim's tennis match if Jim has won 10 out of 25 tennis games so far.

$$\begin{aligned}
 & (1) \\
 & n = \text{number of trials} \\
 & n = 25 \\
 & (2) \\
 & e = \text{number of times an event occurred} \\
 & e = 10 \\
 & (3) \\
 P(e) &= \frac{e}{n} = \frac{10}{25} = 0.4 = \frac{4}{10} = \frac{40}{100} = 40\% \\
 & (4) \\
 & \text{Probability} = 40\%
 \end{aligned}$$

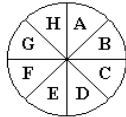
**Step 1:** Identify the value of n.

**Step 2:** Identify the value of e.

**Step 3:** Substitute the known values into the formula and simplify.

Answer: There is a 40% chance that Jim will win his next game.

**Example 2:** Use the spinner to answer the question.



How many times can you expect to spin an A if you spin 8 times?

**Solution:** The number of trials is 8 and the number of possible outcomes is 8 (A-H), but since we only want to spin an A:

$$P(e) = 1/8 \text{ or } 1 \text{ out of } 8 \text{ times.}$$

Answer: You can expect to spin an A one time if you spin 8 times.

**Example 3:** Tammi has passed 6 out of 10 math tests this year. How many more math tests can Tammi expect to pass if there are 25 more tests this year?

$$(1) P = \frac{6}{10} = \frac{3}{5}$$

$$(2) \frac{3}{5} = \frac{x}{25}$$

$$25 \cdot 3 = 5 \cdot x$$

$$\frac{75}{5} = \frac{5x}{5}$$

$$15 = x$$

**Step 1:** Determine the empirical probability of passing a test. Reduce the probability completely. We now know that for every 5 tests, Tammi will pass 3.

**Step 2:** Set up a proportion to determine how many more tests Tammi can expect to pass. In this proportion, the number of tests that are passed is on the top of each ratio and the number of tests taken is on the bottom of the ratio.

Answer: Tammi could expect to pass 15 more tests.

## Probability

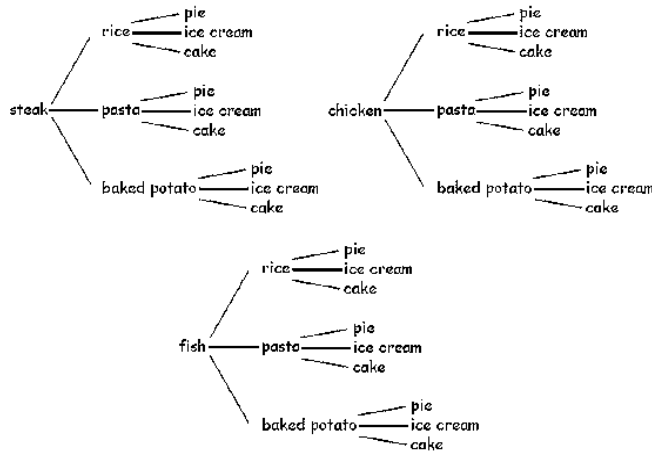
**Probability** is the measure of the chance that a specific outcome will occur. Probability methods at this level include using tree diagrams, sample space, the fundamental counting principle, adding and multiplying probabilities for independent and dependent events, calculating expected value, conditional probability, experimental probability, and theoretical probability.



Tree diagrams are probability tools which represent possible outcomes. If you went to dinner at a banquet, you may be presented with the following possibilities:

- Main dishes: steak, fish, chicken
- Side dishes: rice, pasta, baked potato
- Dessert: pie, ice cream, cake

Suppose that at this dinner you were asked to choose one item from each category: one main dish, one side dish, and one dessert. How many different possible meals could you choose? A tree diagram which gives the sample space (the choices) would help you quickly count the choices:



The above illustration is a tree diagram. All that is left to do is to count the choices down the right side of each branch: there are 27 different possible meals.

Two events are independent if the probability of one event happening has no influence on the probability of the other event happening. If you roll one die and toss one coin, you know that the number on the die has nothing to do with whether the coin toss results in heads or tails. The formula for determining the probability that two independent events will occur is below.

$$P(A \text{ and } B) = \text{Probability of } A \times \text{Probability of } B = P(A) \times P(B)$$

**Example 1:** What is the probability a coin toss resulting in heads and a roll of the die resulting in a 3 or less?

- (1)  $P(A \text{ and } B) = P(A) \times P(B)$
- (2)  $P(A \text{ and } B) = 1/2 \times 3/6$
- (3)  $P(A \text{ and } B) = 1/2 \times 1/2$
- (4)  $P(A \text{ and } B) = 1/4$

Step 1: Choose the correct formula for the probability of A and B happening.

Step 2: There are only two possibilities when tossing a coin (heads and tails), so the probability of a coin toss resulting in heads is 1/2. There are six possibilities when rolling a die (1, 2, 3, 4, 5, and 6). Only three of those possibilities are equal to or less than 3, so the probability of the roll of a die resulting in a 3 or less is 3/6. Substitute the probabilities into the formula.

Step 3: Reduce the fractions before multiplying.

Step 4: 1/2 times 1/2 equals 1/4. Remember to multiply numerators and denominators straight across.

Answer: 1/4

Dependent events are events which influence one another's probability of occurring. The formula for

determining the probability that two dependent events will occur is below.

$$P(A \text{ and } B) = \text{Probability of } A \times \text{Probability of } B \text{ given } A = P(A) \times P(B, \text{ given } A)$$

**Example 2:** If you draw one card from a deck, put it aside, and then draw another card, what is the probability that each card drawn is a heart?

(1)  $P(A \text{ and } B) = P(A) \times P(B, \text{ given } A)$

(2)  $P(A \text{ and } B) = 13/52 \times 12/51$

(3)  $P(A \text{ and } B) = 156/2652$

(4)  $P(A \text{ and } B) = 13/221$

Step 1: Choose the correct formula for the probability of A and B happening.

Step 2: There are 52 cards in a deck of cards. 13 of the cards in each deck are hearts. The probability that the first card drawn is a heart is 13/52. The probability that the second card drawn is a heart is 12/51 because there is one less heart in the deck and one less card in the deck. Substitute the probabilities into the formula.

Step 3: Multiply the fractions. Remember to multiply numerators straight across and denominators straight across.

Step 4: Reduce the fraction completely.

Answer: 13/221

The formula for calculating expected value is:

**(E = result of outcome #1 x probability of a outcome #1 + result of outcome #2 x probability of outcome #2).**

Businesses can use such a formula to roughly project expected profits under specific conditions.

**Example 3:** Suppose you owned a snack bar at a beach. Let's say that in a good summer you make \$3,000 and in a bad summer you lose \$50. The greatest determining factor of a good or bad year has been the weather, and all indications show that the approaching summer season has an 89% chance of being sunny and warm - a good year. What is your projected profit for the approaching season?

(1)  $E = (\$3,000 \times 0.89) + (-\$50 \times 0.11)$

(2)  $E = (\$2,670) + (-\$5.50)$

(3)  $E = \$2,664.50$

Step 1: The result of a good summer is \$3,000 and the probability that there will be a good summer is 89% (0.89). The result of a bad summer is losing \$50 (-\$50) and the probability that there will be a bad summer is 11% (0.11). Use these values to fill in the formula for calculating the expected value.

Step 2: Multiply \$3,000 by 0.89 to get \$2,670 and multiply -\$50 by 0.11 to get -\$5.50.

Step 3: Add the results of Step 2.

The expected profit for the approaching summer season is \$2664.50.

To calculate conditional probability, you must find the probability of an event based on the fact that another event has already happened.

**Example 4:** An algebra class gets a new student, a girl. This new student happens to have two younger siblings. Find the probability that one of the new student's siblings is also a girl.

Solution: Examine all of the possible ways three siblings might be arranged in terms of their gender. The fact that the first sibling, the new girl in class, is a girl alters the possible choices for the problem. The possibilities for the genders of 3 siblings are: GGG (Girl, Girl, Girl), GGB, GBG, GBB, BGG, BGB, BBG, BBB. From these possibilities, you can cancel out any that don't begin with G since we know that the oldest sibling is a girl. That leaves us with 4 possibilities: GGG, GGB, GBG, and GBB. Three of these result in two siblings that are girls. Therefore the probability that at least two of the siblings are girls is  $\frac{3}{4}$  or 75%.

Experimental probability is a way to predict future events using data from past events. Experimental probability is calculated by dividing the number of occurrences of an event by the number of trials of an experiment. A football coach, for example, can predict how well his receiver will complete passes. If the receiver has been completing 10 out of every 25 passes thrown to him, then the coach can use experimental probability to predict how well he will complete passes in the next game:  $\frac{10}{25} = 0.4$  or 40%. The prediction is that the receiver will complete 4 out of 10 or 2 out of 5 passes thrown to him.

In contrast, theoretical probability or mathematical probability refers to finding the probability of an event before any trials of an experiment have been performed. Often theoretical or mathematical probability is referred to as just probability.

If you want to find the probability of rolling a die and getting a 4, you simply set up the fraction  $\frac{1}{6}$  (1 because there is only one 4 on the die and 6 because there are six sides on the die meaning six different possible outcomes.) Therefore, before we even roll a die, we know that theoretical probability tells us that we have a 1 in 6 chance of rolling a 4.

### The Counting Principle

Counting the choices involves determining how many choices are available in a given situation. If there are A choices for one way and B choices for another, then the total number of choices is  $A \times B$ .

**Example 5:** Ana went to the world's largest amusement park. There were 10 different rides, 14 roller coasters, 8 shows, and 6 shops. How many different ways can Ana see all of the attractions?

Solution: Multiply  $10 \times 14 \times 8 \times 6 = 6,720$

Answer: 6,720 ways